$$Q(\beta) = \sum_{i=1}^{n} u_i^2 \qquad ; \; u_i = y_i - \beta_0 - \beta_1 x_i$$

$$\frac{\partial Q(\beta)}{\partial \beta_0} = -2 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i) = 0$$

$$\Leftrightarrow \quad \frac{1}{n} \sum_{i=1}^{n} y_i - \beta_0 - \beta_1 \frac{1}{n} \sum_{i=1}^{n} x_i = 0$$

$$\Leftrightarrow \quad \underline{\underline{\hat{\beta}_0 = \bar{y} - \beta_1 \bar{x}}}$$

$$\frac{\partial Q(\beta)}{\partial \beta_1} = -2 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)(-x_i) = 0$$

$$\Leftrightarrow \quad \frac{1}{n} \sum_{i=1}^{n} y_i x_i - \beta_0 \frac{1}{n} \sum_{i=1}^{n} x_i - \beta_1 \frac{1}{n} \sum_{i=1}^{n} x_i^2 = 0$$

$$\Rightarrow \quad \frac{1}{n} \sum_{i=1}^{n} y_i x_i - (\bar{y} - \beta_1 \bar{x}) \bar{x} - \beta_1 \frac{1}{n} \sum_{i=1}^{n} x_i^2 = 0$$

$$\Leftrightarrow \quad \frac{1}{n} \sum_{i=1}^{n} y_i x_i - \bar{y} \bar{x} = \beta_1 \left(\frac{1}{n} \sum_{i=1}^{n} x_i^2 - \bar{x}^2 \right)$$

$$\Leftrightarrow \quad \hat{\beta}_1 = \frac{\frac{1}{n} \sum_{i=1}^{n} y_i x_i - \bar{y} \bar{x}}{\frac{1}{n} \sum_{i=1}^{n} x_i^2 - \bar{x}^2}$$