Variance formula

Generally, we can write

$$\begin{aligned} Var(X_i) &= E[\,(\; X_i - \mu_X \;)^2\,] \\ &= E[\,(X_i^2 + \mu_X^2 - 2\mu_X X_i)\,] \\ &= E(X_i^2) + \mu_X^2 - 2\mu_X E(X_i) \\ &= E(X_i^2) + \mu_X^2 - 2\mu_X^2 \\ &= E(X_i^2) - \mu_X^2 \equiv E(X_i^2) - E(X_i)^2 \end{aligned}$$

where I denoted $E(X_i) = \mu_X$.

Variance formula

In the special case where X_i is either 0 or 1, we can write

$$\begin{aligned} Var(X_i) &= E[\,(\;X_i - p\;)^2\,] \\ &= E[\,(\;X_i - p\;)^2\,|X_i = 0]\,Pr(X_i = 0) \\ &\quad +E[\,(\;X_i - p\;)^2\,|X_i = 1]\,Pr(X_i = 1) \\ &= (0-p)^2\,(1-p) + (1-p)^2\,p \\ &= p^2\,(1-p) + (1-2p+p^2)\,p \\ &= p^2 - p^3 + p - 2p^2 + p^3 = p - p^2\,. \end{aligned}$$

I denoted $Pr(X_i = 1) = p$. Note that because X_i is either 0 or 1, we know that $E(X_i) = Pr(X_i = 1)$.

Variance formula

Or, you can apply the general formula from above. Note that if X_i is either 0 or 1, we know that $X_i^2 = X_i$ and thus

$$\begin{aligned} Var(X_i) &= E(X_i^2) - E(X_i)^2 \\ &= E(X_i) - E(X_i)^2 \\ &= p - p^2 = p(1-p)\,. \end{aligned}$$

I again denoted $Pr(X_i = 1) = p$ in the last equality.